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# RESEARCH MEMORANDUM

A METHOD FOR ESTIMATING SPEED RESPONSE OF

GAS-TURBINE ENGINES

By Harold Gold and Solomon Rosenzweig

Lewis Flight Propulsion Laboratory Cleveland, Ohio

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

#### RESEARCH MEMORANDUM

A METHOD FOR ESTIMATING SPEED RESPONSE OF

#### GAS-TURBINE ENGINES

By Harold Gold and Solomon Rosenzweig

#### SUMMARY

A brief method is presented for estimating the speed response of turbojet and turbine-propeller engines to a step change in fuel flow. The method approximates the dynamic equilibrium in the gas-turbine engine with a first-order linear differential equation, the time constant of which varies inversely with the equilibrium speed.

For the application of the method to the turbojet engine, the data required are (1) the variation with engine speed of the steady-state engine air flow and the steady-state compressor temperature rise; and (2) the polar moment of inertia of the engine. For the turbine-propeller engine, the variation with engine speed of the steady-state propeller torque is required in addition to the data required for the turbojet engine.

Engine time constants computed by means of this method are compared with time-constant data obtained by direct measurement of transients on three turbojet engines and two turbine-propeller engines. The deviation of the calculated values from the mean experimental values is only slightly greater than the spread of experimental data.

#### INTRODUCTION

Closed-loop automatic control is now being extensively applied to the aircraft gas-turbine engine. As the development of this type of control system for a new engine is often carried on simultaneously with the development of the engine, experimental data on engine dynamic characteristics are not available during the early design stages of the control. Analytical determination of engine dynamic characteristics therefore has become increasingly important.

Experimental studies on turbojet engines have shown that the dynamic equilibrium in a turbojet engine may be closely approximated by a first-order linear differential equation and that during the engine speed transient, the several variables such as temperature, pressure, and air



relation to engine speed. The d

flow, have an essentially linear relation to engine speed. The dynamic characteristics of the turbojet engine may therefore be determined analytically be means of the steady-state thermodynamic relations of the engine and the time constant of the differential equation that expresses the dynamic equilibrium of the engine. A method for obtaining dynamic characteristics from steady-state thermodynamic relations, when the engine time constant is known, is reported in reference 1. The method reported herein and developed at the NACA Lewis laboratory gives a means of estimating the engine time constant from steady-state characteristics that are readily obtainable by thermodynamic analysis.

The method presented herein for estimating the speed response of gas-turbine engines is based on an approximation of compressor and turbine torque transients. Expressions for the engine time constant are derived from this approximation.

A comparison of time-constant values computed by this method with time-constant data obtained by direct measurement of transients on three turbojet and two turbine-propeller engines is presented. The method requires only a brief calculation and appears to be sufficiently accurate for automatic control design.

#### BASIC CONSIDERATIONS REGARDING ENGINE TRANSTENT RESPONSE

Linearity of engine transient response. - Analysis (reference 2) has shown that the torque equilibrium in a jet engine during an acceleration may be expressed by a first-order linear differential equation. This analytical result has been verified by direct measurement of transients on several engines. A typical transient response of a turbojet engine to a step increase in fuel flow rate is shown in figure 1. This response is plotted as the logarithm of the ratio of the difference between initial and final speed to the difference between instantaneous and final speed against time. The data fall quite precisely along a straight line having the equation:

$$t = \tau \ln \frac{N_f - N_1}{N_f - N} \qquad (1)$$

All symbols are defined in appendix A.

Rearranged in the more usual form, equation (1) is

$$N = N_f(1 - e^{-t/\tau}) + N_f e^{-t/\tau}$$
 (la)

The response of a linear first-order system is given by equation (la). This response is described by the engine time constant  $\tau$ .

Transient torque relations. - In the turbojet engine, compressor torque is equal to turbine torque at equilibrium running speed. This equilibrium torque is essentially proportional to the square of the equilibrium speed. The linear response of the engine to a step increase in fuel flow rate indicates that in the transient, the compressor and the turbine torques depart from the equilibrium curve in some manner, such that the unbalanced torque diminishes as a straight line with engine speed. The compatibility of this linear transient with the nonlinear equilibrium torque relation is shown in figure 2. In figure 2 the initial rise shown in turbine torque results from the increase in turbineinlet temperature that accompanies a sudden increase in fuel flow rate. The initial rise shown in compressor torque results from the increase in burner-inlet pressure. The two torque lines are drawn to converge as straight lines to the new equilibrium value. The linearity of these transient torque-speed relations has its basis in: (1) the transient response as characterized by figure 1 and equation (1a); (2) the linearity (during the transient) with speed of the several variables, such as temperature and pressures, as determined by direct measurement during the transient.

In accordance with the torque transient shown in figure 2, the differential equation (derived in appendix B) that expresses the dynamic equilibrium in the engine is

$$IN + (K_C - K_T)N = (K_C - K_T)N_f$$
 (2)

where

 $K_{C}$  slope of compressor transient torque line

 $K_{\mathrm{T}}$  slope of turbine transient torque line

The solution to equation (2) is given by equation (1a) in which the time constant is

$$\tau = \frac{I}{K_{\rm C} - K_{\rm P}} \tag{3}$$

Variation of engine time constant with equilibrium speed. - Variations in engine time constant in a given engine are due to variations in the slopes  $K_{\rm C}$  and  $K_{\rm T}$  as shown by equation (3). These slopes vary over wide ranges with changes in operating level of the engine. Data on several engines show that the value of engine time constant  $\tau$  diminishes as the equilibrium speed increases. The relation between the value of time constant and equilibrium speed appears to be hyperbolic. The difference  $K_{\rm C}$ - $K_{\rm T}$  is then substantially proportional to the equilibrium speed:



$$K_{\mathbf{C}} - K_{\mathbf{T}} = KN_{\mathbf{T}} \tag{4}$$

and therefore

$$\tau = \frac{I}{KN_{T}} \tag{5}$$

Equilibrium shaft torque-speed relation. - The determination of the slopes  $K_{\rm C}$  and  $K_{\rm T}$  by any method will require the determination of the equilibrium value at which the compressor and turbine transient lines converge. For this reason an expression for the equilibrium torque speed relation is derived.

The torque absorbed by any compressor (when specific heat  $c_{\bar{p}}$  is assumed constant and friction is neglected) is proportional to the product of the air flow and the temperature rise divided by the rotational speed. This relation in terms of consistent units is

$$\mathbf{Q}_{\mathbf{C}} = \frac{\mathbf{J}\mathbf{c}_{\mathbf{p}} \ \mathbf{W} \ \Delta \mathbf{T}_{\mathbf{C}}}{2\pi \mathbf{W}} \tag{6}$$

Equation (6) can be rewritten as

$$Q_{C} = \frac{J_{C_{p}}}{2\pi} \left( \frac{W}{N} \right) \left( \frac{\Delta T_{C}}{N^{2}} \right) N^{2}$$
 (6a)

The terms W/N and  $\Delta T_{\rm C}/N^2$  remain substantially constant in a given engine over a wide speed range. The equilibrium shaft torque-speed relation is therefore essentially a squared curve and may be written

$$Q_{C} = K_{C} N^{2}$$
 (7)

where

$$K_{\underline{q}} = \frac{J_{\underline{C}}_{\underline{p}}}{2\pi} \left( \frac{\underline{W}}{\underline{N}} \right) \left( \frac{\Delta T_{\underline{C}}}{\underline{N}^2} \right)$$
 (8)

#### Derivation of Method

Basic approximations. - It has been deduced from experimental observation that the compressor and turbine transient torque-speed lines are essentially straight and that the time constant of the speed



response is essentially inversely proportional to the equilibrium speed. The slopes of the two transient torque lines may therefore be related (equation (4)) and a relation may be written for the intersection of the two lines (equilibrium torque, equation (7)). These relations are insufficient to evaluate the term  $K_C - K_T$ . If, however, the relation of equation (4) can be considered to be valid at all equilibrium speeds, it is necessary to find only one specific configuration of the transient torque lines that can be related to the equilibrium torque relation, in order to obtain a solution for the term  $K_C - K_T$  that will apply at all equilibrium speeds.

As previously discussed, the initial rise in turbine torque results from the increase in turbine-inlet temperature that accompanies a sudden increase in fuel flow rate. The ratio of initial torque rise to the final torque rise (fig. 2) increases as the equilibrium speed is increased. This increase in the torque ratio results from the sharp rise in the magnitude of the fuel flow change for a given increment of speed change at high equilibrium speeds. In general, this ratio will be less than unity for low values of equilibrium speed and greater than unity near maximum engine speed. At some intermediate value of equilibrium speed, the ratio will be unity. In an acceleration to this speed, the slope of the turbine torque line  $K_{\rm TP}$  will be zero.

Empirical relations. - In an acceleration in which the slope of the turbine torque line  $K_{\rm T}$  is zero, it follows from equation (4) that the slope of the compressor torque line  $K_{\rm C}$  is proportional to the equilibrium speed. These conditions are expressed as follows:

$$K_{T} = 0 (9)$$

$$K_{C} = KN_{f} \tag{10}$$

where K is the constant of equations (4) and (5).

As developed in appendix C, the spread of the intercepts (fig. 2) is

$$B - A = KN_{f}^{2}$$

From equation (7)

$$Q_f = K_q N_f^2$$

The constant K may then be related to the constant of the equilibrium torque curve  $\,\mathrm{K}_{_{\mathrm{Cl}}}\,$  by a dimensionless factor.

$$K = nK_{G}$$



The value of the dimensionless constant n has been determined from the measured responses of several engines. The value of n used in this report is unity, which is consistent with all available data.

The value of n=1 fixes the intercept A at the origin of the equilibrium torque line (fig. 2). The final expression for the engine time constant is then

$$\tau = \frac{I}{K_q N_f} \tag{12}$$

Altitude and ram corrections. - From equation (8)

$$K_{q} = \frac{Jc_{p}}{2\pi} \left(\frac{W}{N}\right) \left(\frac{AT_{C}}{N^{2}}\right)$$

The altitude to sea-level correction on the constant  $K_{\rm q}$  is derived from correction on the term  $\left(\frac{W}{N}\right)\left(\frac{AT_{\rm C}}{N^2}\right)$  from which

$$K_{q,corr} = K_{q,a} \frac{\theta}{\delta}$$

This correction and the correction on  $N_{f}$  applied to equation (12) yield the corrected value of time constant

$$\tau_{\rm corr} = \frac{1}{K_{\rm q}} \frac{\delta}{\sqrt{\theta}} \tag{13}$$

This correction is in agreement with that found by the analysis of reference 2. The correction is also verified by experiments in reference 3.

As expressed in equation (8), the effect of ram on the engine time constant is determined by the effect of ram on the term  $\left(\frac{W}{N}\right)\left(\frac{\Delta T}{N^2}\right)$ . This effect may be determined by analysis such as that given in reference 5.

Calculation of turbine-propeller engine time constant. - In the turbine-propeller engine, the equilibrium torque (measured at the turbine) is the sum of the propeller and the compressor torques. The speed response of this type of engine has been found to be essentially linear (references 3 and 5). It can therefore be assumed that the torque transients of the compressor and the turbine are similar to the torque



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transients in the turbojet engine. The steady-state and transient torque-speed relations of the propeller are the same. The transient propeller torque is therefore essentially proportional to the square of engine speed. This transient nonlinearity is not sufficiently large, however, to cause the speed response of the engine to depart from a linear response.

In the application of this method of estimating the speed response to the turbine-propeller type engine, it is considered adequate to approximate the transient torque-speed relation of the propeller by a linear relation; the slope of which is equal to the slope of the equilibrium torque-speed relation at the final speed. The equilibrium torque at fixed blade angle is considered to be proportional to the square of engine speed

$$Q_{D} = K_{D} N^{2}$$
 (14)

The slope of the torque-speed relation at the final speed  $\left(\frac{\partial Q}{\partial N}\right)_{\beta}$  is then  $2K_p$   $N_f$ .

On the basis of these approximations, the differential equation (derived in appendix B) that expresses the dynamic equilibrium in the engine is

$$IN + (K_C + 2K_p N_f - K_T)N = (K_C + 2K_p N_f - K_T)N_f$$
 (15)

The solution to equation (15) is given by equation (1a) in which the time constant is

$$\tau = \frac{I}{(K_C + 2K_p N_f - K_T)}$$
 (16)

In accordance with the approximation of this method, when

$$K_T = 0$$

$$K_C = K_q N_f$$

When these relations are substituted in equation (16) the relation reduces to

$$\tau = \frac{I}{(K_{q} + 2K_{p})N_{f}}$$
 (17)



Comparison of measured and calculated time constants. - Results of computations of engine time constant made by the method described herein and the mean experimental values for three turbojet engines are presented in figure 3. (A sample calculation is presented in appendix C.) The equilibrium shaft torque and engine speed curve of the engine is shown in the upper part of figure 3. The circled points shown on the equilibrium torque plot are the values computed from steady-state measurement of compressor air flow, temperature rise, and speed. The curve shown on the equilibrium torque plot is the mean squared torque-speed relation based on the values of the circled points. The torque points are in good agreement with the squared curve. The solid curve shown on the time constant plot is the hyperbola computed by means of equation (12) from the mean squared torque curve. The circled points are the time constant values computed from the circled equilibrium torque points. In the case of the circled time constant points, the squared equilibrium torque curve is therefore considered to pass through the particular equilibrium torque point. This procedure may be of value in regions where the equilibrium torque curve deviates appreciably from a consistent squared relation. The dashed line shown on the time constant plot is the average experimental value. The data from which the mean experimental curves were drawn showed a spread of approximately ±20 percent. The maximum deviation of the calculated values is between 25 and 30 percent. In the three comparisons of experimental and calculated values, the calculated values fall both above and below the experimental values. It is on this basis that the value of unity for the dimensionless constant n was derived.

Results obtained for two turbine-propeller engines are presented in figure 4. A comparison of measured and computed time constants for three blade-angle settings is shown in figure 4(a) for turbine-propeller engine A. The effect of blade angle on the computed values of time constant is greater than the effect indicated by the experimental values. The deviation of the calculated values from the mean experimental values is, nevertheless, approximately the same as was indicated for the turbojet engines.

The agreement between computed values of time constant and a measured value on a second turbine-propeller engine is shown in figure 4(b) for turbine-propeller engine B. In this instance, the experimental value was obtained by the frequency-response technique. The data on all the other engines were obtained by the step technique.

Similarity of engine time constants. - The values of engine time constant for the three turbojet engines shown in figure 3 are nearly equal at maximum engine speed. It is noteworthy that this equality

exists in the face of substantial differences in the inertia, the torque, and the speed range of these engines. The values of these parameters are listed in the following table.

Turbojet engine	Polar moment of inertia I (1b-ft sec <sup>2</sup> )	speed N	Torque at maximum speed Q (lb-ft)	maximum	1	Computed time constant at maximum speed (sec)
A	16.30	7,794	8955	14.16	1.9	1.4
В	2.40	12,500	2192	13.68	1.5	1.6
C	5.42	11,500	4410	14.15	1.2	1.5

Transients of large magnitude. - The values of engine time constant computed by the method of this report are independent of the magnitude of the transient and are a function only of the equilibrium speed. The experimental data from which the time constant curves of figures 3 and 4 were drawn were obtained with small changes in fuel flow rate. Data available at present are insufficient to determine either the limit of linearity of engine response or the dependence of the time constant on equilibrium speed alone during transients of large magnitudes.

#### CONCLUDING REMARKS

Experimental data indicate that the speed response of turbojet and turbine-propeller engines is essentially linear and first order and therefore the concept of time constant may be used to describe the speed response of such engines. Also the value of the time constant is indicated to be inversely proportional to the equilibrium speed.

An analysis, based on the concept of an equilibrium shaft torque that is proportional to the square of engine speed, shows that the constant of the equilibrium torque-speed relation may be related to the constant of the time constant-speed relation. Experiment shows that these two constants may be equated. The engine time constant may therefore be computed from the steady-state shaft torque-speed relation of the engine.

A comparison of engine time constants computed by this method with time constant data obtained by direct measurement of transients on three turbojet and two turbine-propeller engines shows that the deviation of the calculated values from the mean experimental values is only slightly greater than the spread of experimental data.

Lewis Flight Propulsion Laboratory
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## APPENDIX A

## SYMBOLS

The following symbols have been used in this report:

A,B,C,K,N	constants ,				
c <sup>p</sup>	average specific heat of gas passing through compressor (assumed to be 0.243 Btu/(lb)(°F))				
I	polar moment of inertia of entire engine (related to engine speed), lb-ft $\sec^2$				
J	mechanical equivalent of heat, 778 ft-lb/Btu				
N	engine speed, rpm				
Ň	time derivative of engine speed				
Q	torque, lb-ft				
T	temperature, <sup>O</sup> F				
t	time, sec				
	temperature rise, <sup>O</sup> F				
W	engine air flow, lb/sec				
δ	ambient static pressure  NACA standard sea-level pressure				
θ	ambient static temperature  NACA standard sea-level temperature				
τ	engine time constant, sec				
Subscripts:	, and the second				
a	alțitude				
C	compressor				
corr	corrected				
f	final				

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i	initial	
p	propeller	· - :- <del></del>
ď.	compressor (when applied to equilibrium torque constant)	23
<b>T</b> .	turbine	<u></u> 35
(t)	denoting instantaneous value during transient	·
β	blade angle	viv let

#### APPENDIX B

#### DERIVATION OF DIFFERENTIAL EQUATIONS EXPRESSING

### DYNAMIC EQUILIBRIUM

Turbojet engine (equation (2)). - In accordance with figure 2, the instantaneous values of compressor and turbine torques during the speed transient may be written:

$$Q_{C(t)} = K_C N + A$$
 (B1)

$$Q_{T(t)} = K_T N + B$$
 (B2)

when

$$N = N_{f}$$

$$Q_{C(t)} = Q_{T(t)} = Q_{f}$$

substitution of these values in equations (B1) and (B2) yields

$$A = Q_{f} - K_{C} N_{f}$$
 (Bl(a))

$$B = Q_f - K_T N_f$$
 (B2(b))

substitution again in equations (Bl) and (B2) gives

$$Q_{C(t)} = K_{C} N + Q_{f} - K_{C} N_{f}$$
 (B3)

$$Q_{T(t)} = K_{T} \dot{N} + Q_{f} - K_{T} N_{f} \qquad (B4)$$

The dynamic equilibrium during the transient is

$$\dot{IN} = Q_{T(t)} - Q_{C(t)}$$
 (B5)

Substitution of equations (B3) and (B4) in (B5) and rearranging terms gives

$$I\dot{N} + (K_{C} - K_{T})N = (K_{C} - K_{T})N_{T}$$
 (B6)

Turbine-propeller engine (equation (15)). - The compressor and the turbine torque transients are assumed to be the same in this type of engine as in the turbojet engine. The instantaneous values of compressor

and turbine torque are therefore expressed by equations (B1) and (B2), respectively. The transient torque-speed relation of the propeller is approximated by a linear relation, the slope of which is equal to glope of the equilibrium torque-speed relation at the final speed. The equilibrium torque-speed relation at fixed blade angle is approximated by a squared relation. Therefore

$$Q_p = K_p N^2$$

The slope of the linear relation that approximates the transient is

 $\left(\frac{9N}{96^{b}}\right)^{\beta} = 5K^{b} N^{\xi}$ 

The linear torque-speed relation is then

$$Q_{p}(t) = (2K_{p}N_{f})N + C$$

When  $N = N_{f}$ 

$$Q_{p(t)} = Q_{pf}$$

$$Q_{T(t)} = Q_{Tf}$$

$$Q_{c(t)} = Q_{cf}$$

Substituting these values in equations (B1), (B2), and (B7) gives

$$A = Q_{Cf} - K_C N_f$$

$$B = Q_{Tf} - K_T N_f$$

$$C = Q_{pf} - (2K_p N_f)N_f$$

Substituting again in equations (Bl), (B2), and (B7) yields

$$Q_{c(t)} = K_c N + Q_{cf} - K_c N_f$$
 (B8)

$$Q_{T(t)} = K_T N + Q_{Tf} - K_T N_f$$
 (B9)

$$Q_{p(t)} = (2K_p N_f)N + Q_{pf} - (2K_p N_f)N_f$$
 (Blo)

The dynamic equilibrium during the transient is

$$\dot{IN} = Q_{T(t)} - Q_{C(t)} - Q_{p(t)}$$
 (BL1)

Substituting equations (B8), (B9), and (Bl0) in equation (Bl1) and employing the relation

$$Q_{Tf} = Q_{Cf} + Q_{pf}$$

yield

$$\dot{N} + (K_C + 2K_p N_f - K_T)N = (K_C + 2K_p N_f - K_T)N_f$$
 (Bl2)

### APPENDIX C

#### RELATION BETWEEN INTERCEPTS OF TRANSIENT

#### TORQUE LINES

From equations (Bl(a)) and (B2(a)) of appendix B

$$K_{C} = \frac{Q_{f}}{N_{f}} - \frac{A}{N_{f}}$$
 (C1)

$$K_{\underline{T}} = \frac{Q_{\underline{f}}}{N_{\underline{f}}} - \frac{B}{N_{\underline{f}}}$$
 (C2)

From these expressions

$$K_{C} - K_{T} = \frac{B-A}{N_{f}}$$
 (C3)

Combining this relation with equation (4) gives

$$B - A = KNf2$$
 (C4)

#### APPENDIX D

#### SAMPLE TIME-CONSTANT CALCULATIONS

<u>Turbojet engine</u>. - The steady-state engine data for the turbojet engine are:

The polar moment of inertia I of the engine is 5.43 pound-feet per second per second.

According to equation (6) the equilibrium shaft torque is

$$Q_{C} = 1811 \frac{W \Delta T_{C}}{N} lb-ft$$

$$Q_{C} = \frac{1811 \times 80 \times 350}{11,500} = 4410 \text{ lb-ft}$$

From equation (7)

$$K_{q} = \frac{Q_{C}}{N^{2}} \left(\frac{60}{2\pi}\right)^{2} \text{ lb-ft sec}^{2}$$

$$K_{q} = \frac{4410}{(11,500)^{2}} \times \left(\frac{60}{2\pi}\right)^{2} = 3.05 \times 10^{-3} \text{ lb-ft sec}^{2}$$

From equation (12)

$$\tau = \frac{I}{K_{q}} \frac{1}{N_{f}} \left( \frac{60}{2\pi} \right) \sec \alpha$$

$$\tau = \frac{5.42}{3.05 \times 10^{-3}} \frac{1}{N_{\rm f}} \left( \frac{60}{2\pi} \right)$$

$$\tau = \frac{17,000}{N_P} \sec$$

at  $N_p = 11,500 \text{ rpm}$ 

$$\tau = \frac{17,000}{11,500} = 1.48 \text{ sec}$$

<u>Turbine-propeller engine</u>. - The steady-state engine data for the turbine-propeller engine are:

The combined polar moment of inertia I of engine and propeller is 10.68 pound-feet per second per second.

From equation (6)

$$Q_{\rm C} = \frac{1811 \times 51.39 \times 398}{8040} = 4600 \text{ lb-ft}$$

From equation (7)

$$K_q = \frac{4600}{(8040)^2} \left(\frac{60}{2\pi}\right)^2 = 6.48 \times 10^{-3} \text{ lb-ft sec}^2$$

From equation (14)

$$K_{p} = \frac{Q_{p}}{N^{2}} \left(\frac{60}{2\pi}\right)^{2} \text{ lb-ft sec}^{2}$$

$$K_p = \frac{1920}{(8040)^2} \left(\frac{60}{2\pi}\right)^2 = 2.7 \times 10^{-3} \text{ lb-ft sec}^2$$

$$(K_q + 2K_p) = (6.48 + 5.4) \times 10^{-3} = 11.88 \times 10^{-3} \text{ lb-ft sec}^2$$

From equation (17)

$$\tau = \frac{I}{(K_q + 2K_p)} \frac{1}{N_f} \left(\frac{60}{2\pi}\right) \sec$$

$$\tau = \frac{10.68}{11.88 \times 10^{-3}} \frac{1}{N_{\rm f}} \left(\frac{60}{2\pi}\right) \sec$$

$$\tau = \frac{8600}{N_{\text{f}}} \, \text{sec}$$

at  $N_f = 8040$ 

$$\tau = \frac{8600}{8040} = 1.07 \text{ sec}$$

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  - 5. Oppenheimer, Frank L., and Jacques, James R.: Investigation of Dynamic Characteristics of a Turbine-Propeller Engine. NACA RM E51F15, 1951.

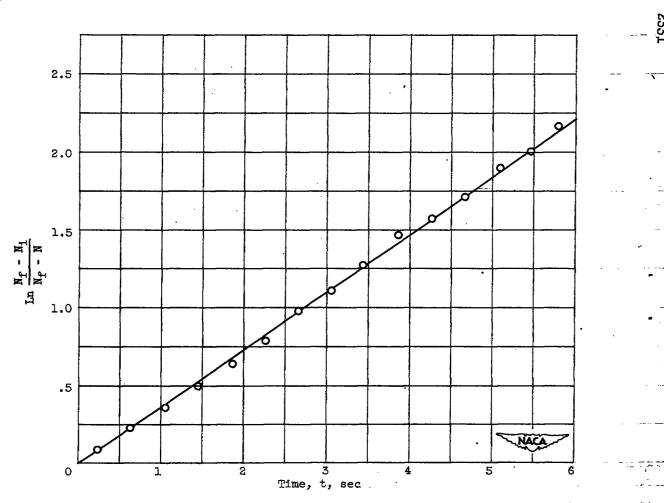


Figure 1. - Semilogarithmic plot of typical turbojet engine transient. Response of engine speed to step increase in fuel flow.

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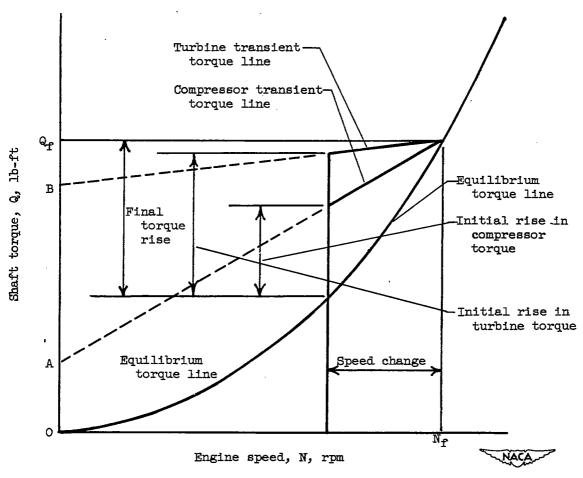
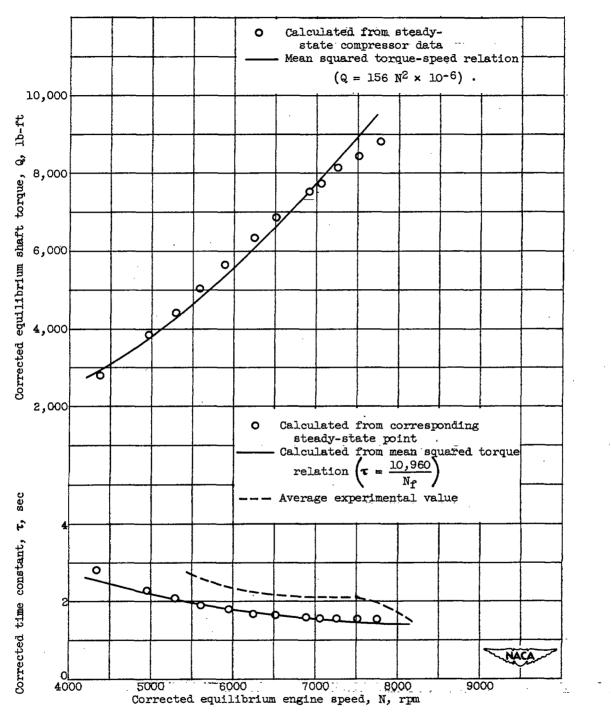


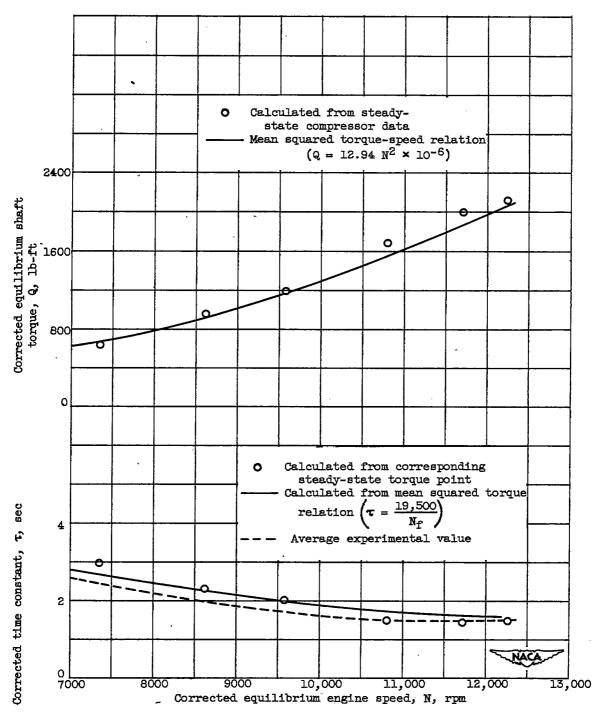
Figure 2. - Representative torque variations during turbojet engine speed transient.





(a) Turbojet engine A; polar moment of inertia 16.30 pound feet per second per second.

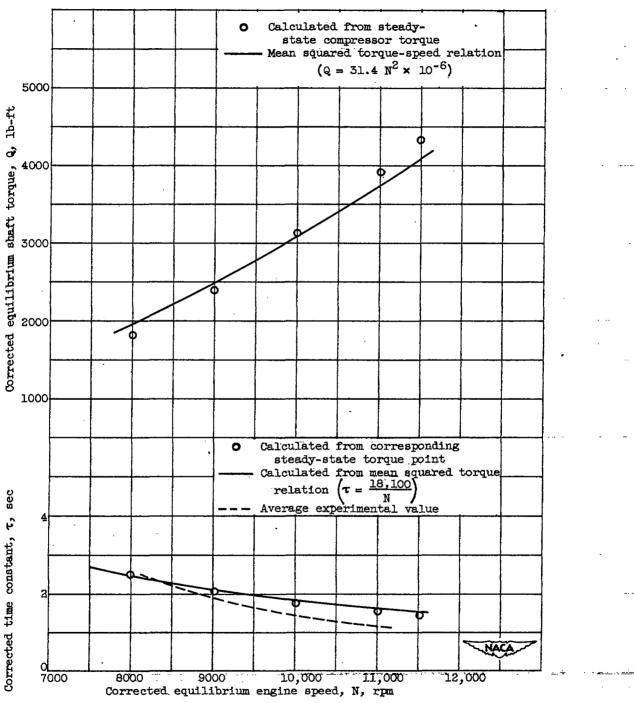
Figure 3. - Variation of equilibrium shaft torque and engine time constant with engine speed in turbojet engine at sea-level static conditions.



(b) Turbojet engine B; polar moment of inertia 2.40 pound feet per second per second.

Figure 3. - Continued. Variation of equilibrium shaft torque and engine time constant with engine speed in turbojet engine at sea-level static conditions.



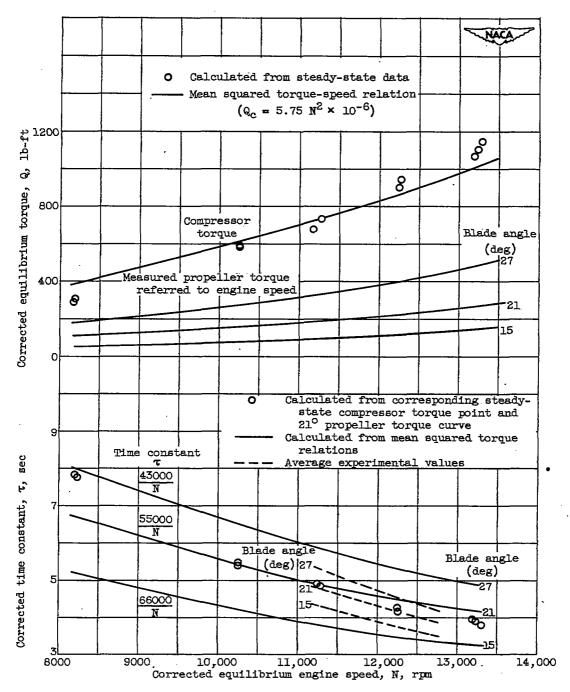


(c) Turbojet engine C; polar moment of inertia 5.42 pound feet per second per second.

Figure 3. - Concluded. Variation of equilibrium shaft torque and engine time constant with engine speed in turbojet engine at sea-level static conditions.

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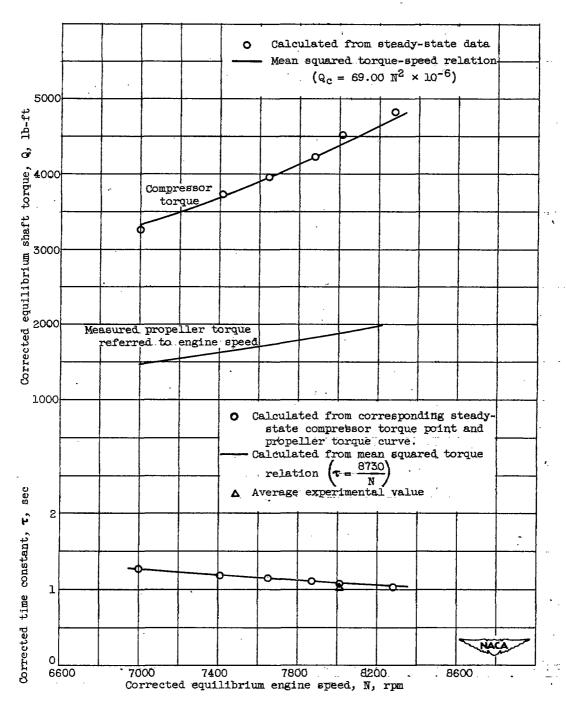




(a) Turbine-propeller engine A; for three blade angles; combined polar moment of inertia of engine and propeller, 4.55 pound feet per second per second.

Figure 4. - Variation of compressor and propeller equilibrium torque and engine time constant with engine speed in turbine-propeller engine at sea-level static conditions.





(b) Turbine-propeller engine B, blade angle, 35.4°, combined polar moment of inertia of engine and propeller, 10.68 pound feet per second per second.

Figure 4. - Concluded. Variation of compressor and propeller equilibrium torque and engine time constant with engine speed in turbine-propeller engine at sea-level static conditions.



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1.1.7.1% (17%・機能) \* **学院小選問題** 

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